PROCEDURAL AND CONCEPTUAL KNOWLEDGE IN CALCULUS BEFORE ENTERING THE UNIVERSITY: A COMPARATIVE ANALYSIS OF DIFFERENT DEGREE COURSES

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The role of mathematics as a subject in tertiary education differs enormously among various degree courses. For Natural Sciences, mathematics is an important tool for every student. In economics, its role depends on the areas of interest, whereas in physics and mathematics, it is in the core of the study programmes. In all of these courses, a particular emphasis is put on calculus. A testing instrument is presented for students' procedural and conceptual knowledge in calculus at the end of their school careers, based on German common core standards covering three areas of procedural knowledge and one area of conceptual knowledge. In a survey with 1134 students of different degree programmes, students' knowledge is compared. Finally, it is investigated as to which dimension best describes competencies in calculus.

THE CHALLENGES IN UNIVERSITY DEGREE COURSES

The increase in the numbers of students who successfully complete degree courses in the STEM academic disciplines of science, technology, engineering, and mathematics is a declared goal in many countries. The widespread efforts to attract more students to start their careers go along with problems of those who have opted to do so. Recent studies have made it clear that dropouts remain as a significant problem in different countries. See, for instance, Chen (2012) for the case of colleges in the United States and Dieter (2011), who examines degree courses in mathematics in Germany.

The changes in school mathematics over the last two decades have not brought many changes to the problem that mathematics remains a challenge in all degree courses at colleges and universities. Hoyles, Newman, and Noss (2001) even claim that the shift towards utilitarian mathematics makes the situation rather more difficult. This also involves the area of calculus (Ganter, 2000), which is traditionally important in the beginning of tertiary education in mathematics because sciences and the economy make frequent use of it. Lately, studies have indicated that motivational aspects are crucial for successful completion of calculus courses at colleges (Pyzdrowski, 2013). Interestingly, interventional studies at Colorado State University (Pilgrim, 2010) have shown no significant differences concerning epistemological beliefs using the Modified Indiana Mathematical Belief Scales, between the students who passed the calculus exams easily, and participants in an intervention course for those who were at risk of failing the exams.

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PROCEDURAL AND CONCEPTUAL KNOWLEDGE IN CALCULUS

As it is a broad field, knowledge in calculus has to be built up slowly and over a long period of time. Various learning theories describe the cumulative nature of building up knowledge in calculus, and it is not an easy endeavour to compare results that were obtained in different theoretical settings. Among the various systems for describing knowledge in calculus, the distinction between procedural and conceptual knowledge (Hiebert, 1986) is tried and tested in calculus (See e.g., Porter & Masingila, 2000).

Procedural knowledge is defined as action sequences for solving problems, whereas conceptual knowledge aims at "explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain" (Rittle-Johnson & Alibali, 1999, p. 175). Star and Stylianides (2013) argue theoretically that there has to be a gap between procedural and conceptual knowledge. It is one of our aims in this study to better understand to what extent procedural and conceptual knowledge differ in calculus even before entering the universities.

ACHIEVEMENTS IN CALCULUS AT SCHOOL

For the understanding of the design of this study, please note that the data were gathered at a university in Germany, where calculus is compulsory for all high school students. Education standards in Germany established a consensus among the conference of ministers of education of the federal states with the aim to improve school education. One aim of the standards was to provide a theoretical framework that allows students to gain competencies that can be measured empirically (Ehmke, Leiß, Blum, & Prenzel, 2006).

National standards for students to pass final school exams and qualify for entering universities (Kultusministerkonferenz, 2012) are the bases for the core curricula in the federal states. The standards distinguish between comprehensive mathematical competencies (arguing mathematically, mathematical problem solving, mathematical modelling, using mathematical representations, and being in command of symbolic, formal, and technical elements of mathematics) and content-related competencies, following the "guiding ideas" of "algorithm and number", "measuring", "room and shape", "functional relations", and "data and chance". The core curriculum of the state of Lower Saxony (Niedersächsisches Kultusministerium, 2009), in which about 70% of all participants of this survey have passed their final school exams, is quite compatible with this system.

Since calculus plays an important role in universities, often in special courses, this project aims at looking at different degree courses in a much more detailed way. Our longitudinal study on different areas of mathematics (Halverscheid & Pustelnik, 2013) compared competencies of students of physics and mathematics on entering the university and their exam results in the first courses. This project concentrates on the calculus and aims at considering both procedural knowledge and conceptual knowledge. For this aim, the competencies named in the federal core curriculum are considered in four different areas. It should be stressed that the underlying theories for mathematical competencies on the one hand and the difference between conceptual and procedural knowledge on the other hand are not exchangeable. What we did was to classify the competencies according to three areas of procedural knowledge and one area of conceptual knowledge as shown in Table 1.

	Area	Competencies according to the federal core curriculum. Students…
T	Procedural knowledge on calculating derivatives	… compute derivatives for the following classical functions with the rules of sums, products, factors, and composition: polynomials, sin, $\sqrt{\ }$, exp, and compositions of these
S	Procedural knowledge on curve sketching	…determine slopes of tangents to graphs …search extremal points and inflexion points with derivatives …use derivatives to discuss monotonicity and curvature, investigate extremal points, and analyse functions defined by sections
I	Procedural knowledge on integration	…compute integrals with the help of antiderivatives of polynomials, sin, $\sqrt{\ }$, exp, $x \mapsto x^z, z \in \mathbb{Z}$, …reconstruct graphs of a function from that of its derivative and vice versa …interpret the integral as an area and reconstructed stock …illustrate the main theorem of calculus for the graph, the function, and its derivative
C	Conceptual knowledge on differential calculus	…use pre-concepts of limits for differentiation and integration … use different classes of functions and compositions of them to describe functional phenomena and to solve inner- and outer-mathematical problems …describe and interpret rates of growth functionally …explain rates of growth …interpret derivatives as rates of growth …employ models of limited and logistic growth

Table 1: Grouping of competencies according to the national standards (in Germany)

RESEARCH QUESTIONS

On the one hand, we expected higher conceptual abilities of students in Physics and Mathematics. On the other hand, these are the subjects with no limited access at all, i. e. everyone with a successful final school exam may enrol in physics and mathematics.

The area of techniques should not be too difficult for either of the degree courses. In all of the courses, many students should be able to answer many questions correctly, and the differences between the degree courses should be smaller than the differences between the areas.

For the problem of how to deal with the difficulties of students in their first academic year, it would be important to know how heterogeneous the groups of the degree courses are.

The following questions served as guiding lines for this research project:

To what extent do the attendants of the degree courses enter the university with different prerequisites concerning calculus?

Can differences within a single degree course be detected?

Are there characteristic differences between the areas of procedural and conceptual knowledge? Is it possible to develop a high standard of conceptual knowledge in calculus while having less elaborate procedural knowledge?

TEST DESIGN

For each of the areas (T), (S), (I), and (C), 15 items were constructed in such a way that to every competence at least two items correspond.

To illustrate the test design, we give a couple of examples for the listed competencies. In area (T), the item

"A function is given by $f(x) = \sin(a \cdot x + b)$. Compute its derivative. Mark the correct answer:

☐ $f'(x) = \sin(a)$ ☐ $f'(x) = \cos(a)$ ☐ $f'(x) = a \cdot \sin(a \cdot x + b)$

☐ $f'(x) = a \cdot \cos(a \cdot x + b)$"

is relevant for the competence to "compute integrals with the help of antiderivates of sin".

The following item refers to the first competence in area (S), "procedural knowledge on curve sketching".

"The function defined by $f(x) = x^2$ has the derivative$f'(x) = 2x$, which assertions on the monotonicity properties of f hold? Mark the correct answers.

☐ f is strictly monotonically increasing on all of $\mathbb{R}$.

☐ f is strictly monotonically increasing on.$\mathbb{R}_0^+$.

☐ f is strictly monotonically decreasing on all of $\mathbb{R}$.

☐ f is strictly monotonically decreasing on $\mathbb{R}_0^-$.

☐ f is monotonically increasing on the interval, $[-1; 1]$.

☐ f is monotonically increasing the interval, $[1; 2]$."

The following item concerns the first competence in area (I).

"Compute the integral$\int_{-1}^{3} \frac{1}{2} x^2 dx$ and mark the correct result.

☐ $\frac{4}{5}$ ☐ $\frac{5}{6}$ ☐ $\frac{6}{5}$ ☐ $\frac{14}{5}$ ☐ $\frac{7}{2}$ ☐ $\frac{14}{3}$."

Finally, consider the following example for conceptual knowledge, area (C), for the competence to "describe and interpret rates of growth functionally":

"At a and b, the graph of a function f has a horizontal secant. Which of the following assertions on f is true?

☐ f is on the interval constant.

☐ f is on the interval monotonically increasing or monotonically decreasing.

☐ f is a linear function.

☐ None of the above holds."

PARTICIPATING STUDENTS AND RELEVANT DEGREE COURSES

In this university, mathematics is compulsory in the degree courses of Natural sciences, Agriculture, Forestry, Economy, Computer Science, and of course, Mathematics itself. For the Natural sciences, Economy, Computer Science, and Mathematics, this involves both degree courses with one major and degree courses for teacher education with two subjects of equal weight. To ease the transition from school to tertiary education, a system of preparatory courses is offered to students in four clusters. The corresponding degree courses are listed here jointly with the numbers of participants in the tests. Economy: 494 participants; Physics, Computer Science and Mathematics: 195 participants; Geology and Biology: 131 participants; Agriculture and Forestry: 314 participants.

METHODOLOGY

For each of the test sections a one-dimensional Rasch analysis was conducted, so every person was assigned one parameter per section. To gain questionnaires satisfying the Rasch model, some of the items had to be eliminated. Finally, the section on Calculating derivatives contains eight items, the section on Curve sketching contains eleven items, the section on Integration contains six items, and the section on Conceptual knowledge on differential calculus contains seven items.

Since abilities for a person answering every item correctly or answering every item incorrectly cannot be estimated, the number of persons per test section had to be reduced. Some data has also been excluded due to some participants not filling out all of the questionnaire. Overall between 413 and 896 participants are part of the analysis.

RESULTS

To answer the first research question the mean values of person parameters were calculated for each preparation course, which can be seen in Table 2. The four Rasch models were scaled such that the mean values for participants of the degree courses in Agriculture and Forestry were 0. On the whole, the use of IRT methods has led to convincing results. However, the personal parameter estimation for Agriculture /

Forestry should considered carefully in the areas (I) and (C), where 90 % of the students did not give any correct answer at all.

While the differences between Mathematics and the other three courses are highly significant (p<0.01) for all of the test sections, the differences between Economics and Geology/ Biology are not significant in all cases (p>0.05). The differences between Agriculture/Forestry and Economics respectively Geology/ Biology are also significant for two sections: Calculating derivatives and Curve sketching. The variances of the four courses can also be seen in Table 2.

	Economy	Mathematics, Computer Science, Physics	Geology, Biology	Agriculture, Forestry
Integration	0.25/ 0.81	1.29/ 1.52	0.38/ 0.65	0/ 0.48
Curve sketching	0.32/ 0.90	2.22/ 1.33	0.34/ 1.13	0/ 0.78
Calculating derivatives	0.79/ 1.61	2.47/ 0.77	0.75/ 2.00	0/ 1.67
Conceptual knowledge	-0.04/ 0.75	1.54/ 1.66	0.16/ 0.86	0/ 0.79

Table 2: Mean Values and Variances of person parameters for each test section

The effect size of these differences is strong in comparing Mathematics and the three other degree courses for all sections (d>0.8). The two significant differences between Agriculture/ Forestry and Economics and Geology/ Biology are of small size (d>0.4).

Whereas the variances are the highest for Mathematics in three of the areas, they are the smallest in Calculating derivatives. The ratio of variances differs from 1 for area (T) and area (I) is highly significant (p<0.001) for differences between Mathematics and the other three degree courses, whereas other differences are not significant.

To investigate the reason of the small variance in Calculating derivatives for students of Computer Science, Physics, and Mathematics, we look at the quartiles of the distribution of the person parameters. About 25% of these students answered every item in the Calcuating derivatives test correctly, and were not estimated by the IRT method. Half of the remaining students had only one item wrong. So the small variance seems to be due to a ceiling effect.

DISCUSSION

With respect to the first two research questions, we can see that the students in the Computer Science, Physics, and Mathematics degree courses show by far the best results in every area of Calculus. This group of students possesses the highest mean values in every section. The students of Economy and Geology and Biology have mean values being nearly the same for every section and students in Agriculture and Forestry have the lowest values besides conceptual knowledge.

The other courses show results with smaller differences. While students in Agriculture and Forestry have the weakest results in three out of four areas, the differences have only small effect sizes. No significant differences between students in Economy and Geology/ Biology can be established.

Finally, solid conceptual knowledge occurs only in exceptional cases apart from in Computer Science, Physics, and Mathematics. And even in that group, there is a dichotomy between those with good conceptual knowledge and those who answer only a small part of these questions. In their empirical study on children's conceptual understanding of mathematical equivalence, Rittle-Johnson & Alibali's (1999) findings, "suggest that conceptual knowledge may have a greater influence on procedural knowledge than the reverse". One might see the results of this survey as supportive of this claim for the case of calculus in as far as those with a strong conceptual knowledge also did very well on the procedural knowledge of calculus.

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