The dynamical origin of Hawaiian volcanism

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Abstract

We study the dynamics of melting in the Hawaiian plume using a 3D variable-viscosity convection model outfitted with a melting parameterization that permits calculation of the local melting rate and the distribution of buoyant depleted residual material. From a suite of 45 steady-state numerical experiments, we derive complete scaling laws for the total rate of melting $M$ and the height $H$ and width $W$ of the topographic swell as functions of the lithospheric thickness $z_l$ and the plume’s maximum potential temperature $\theta_i$, thermal buoyancy flux $B$, and minimum viscosity $\eta_p$. Assuming $1500 ^\circ C < \theta_i < 1600 ^\circ C$, the observed values of $M$, $H$ and $W$ can only be matched if $z_l \leq 89$ km, $2200 \text{ kg s}^{-1} \leq B \leq 3500 \text{ kg s}^{-1}$, and $\eta_p \geq 5 \times 10^{17} \text{ Pa s}$. We study a reference Hawaiian model satisfying these constraints. The depletion anomaly is narrower than the thermal anomaly, and carries 24% of the total (thermal plus depletion) buoyancy flux. Its buoyancy contributes 350 m of the uplift along the swell axis, and reduces the geoid/topography ratio by 16% relative to a model without depletion buoyancy. All the material that melts comes from the hottest central part of the plume, and no direct melting of the asthenosphere or lithosphere occurs. Melting occurs both in a primary melting zone above the plume stem and in a weaker secondary melting zone 300–500 km downstream, separated by an interval where no melting occurs. We propose that the preshield-, shield-, and postshield stages of Hawaiian volcanism are generated by the primary melting zone, and the rejuvenated stage by the secondary melting zone.

Keywords: volcanism; melting; viscosity; heat transfer; mantle plumes; buoyancy

1. Introduction

The Hawaiian islands are the most spectacular example of intraplate volcanism produced by an upwelling mantle plume. The presence of the Hawaiian plume is manifested by surface signatures with a diversity of spatial scales. The largest scales (1000 km or more) are represented by the Hawaiian swell, a topography anomaly some 1200 km wide and 3000 km long, and by its associated geoid anomaly. Signatures at intermediate scales (a few hundred km) include the width of the volcanically thickened crust (~250–300 km), the flexural arch produced by loading of the elastic lithosphere, and the temporal variations of eruption rate and lava composition at individual Hawaiian volcanos, which occur over a time corresponding to a few hundred km of motion of the Pacific plate over the hotspot. Signatures at the smallest scales (<100 km) include the characteristic spacing of the islands and the en echelon pattern of the volcanic centers [1].

All these surface signatures are either produced or significantly affected by melting in the sub-lithospheric mantle. In this paper and a companion
(Cserepes et al., [2]), we seek to explain the most important of these signatures using a self-consistent dynamical model that includes the effects of melting.

The principal difficulty in modeling the Hawaiian plume is the inherent three-dimensionality of the flow due to the motion of the Pacific plate. The first models to incorporate this effect were either kinematic [3] or based on lubrication theory [4], while more recent models include heat transport and variable viscosity [5,6]. However, none included the effects of melting.

Melting has several effects on mantle flow. Most importantly, absorption of latent heat reduces the temperature, and consequently also the thermal buoyancy, of the ascending rock [7]. However, the effective buoyancy is simultaneously increased by the presence of interstitial melt (‘melt retention buoyancy’ [8]) and by removal of denser mineral phases from the residual solid (‘depletion buoyancy’ [9]). These effects have been extensively studied for mid-ocean ridges ([10] and references therein), but less so for mantle plumes. Watson and McKenzie [11] used an axisymmetric, constant-viscosity model to estimate the distribution of melting in the Hawaiian plume, including the effect of latent heat on the temperature but ignoring all effects of melting on buoyancy. Olson [12] and Farnetani and Richards [13,14] used similar models to study melting in unsteady starting plumes. Phipps Morgan et al. [15] used lubrication theory to study the role of depletion buoyancy in generating hotspot swells, but did not determine temperature or melting rate fields. More rigorous studies of the role of depletion buoyancy in plume dynamics have been limited to two dimensions [16,17].

In this study we address two questions. First, how much does depletion buoyancy contribute to the Hawaiian swell and its associated geoid anomaly? We focus on depletion because its effect on density (a decrease of 0.7% for 10% depletion [18]) is several times larger than the effects of melt retention or latent heat absorption [19]. Second, we shall investigate how the spatial distribution of the melting rate within the plume is related to the temporal evolution of volcanism, and propose a new dynamical explanation for rejuvenated volcanism in the Hawaiian islands.

Fig. 1. Sketch of the numerical model. A rectangular box of depth \( d = 400 \) km contains variable-viscosity fluid. Shear flow is driven by an imposed horizontal velocity \( U \). A thermal plume is generated by a potential temperature anomaly of amplitude \( \Delta \theta \) on the bottom of the box. Additional boundary conditions are discussed in [5]. Depth coordinate \( z \) increases downward from the upper surface, and \((x, y) = (0, 0)\) is above the center of the plume at \( z = d \).

2. Numerical model

The model geometry and boundary conditions are shown in Fig. 1. A box of depth \( d = 400 \) km and adjustable length and width contains fluid whose viscosity \( \eta \) varies with depth \( z \) and potential temperature \( \theta \) as:

\[
\eta = \eta_0 \exp \left( \frac{E + \rho_0 g z V}{R \theta} - \frac{E + \rho_0 g d V}{R \theta_0} \right) \tag{1}
\]

where \( g \) is the gravitational acceleration, \( R \) is the gas constant, \( \eta_0 \approx 10^{21} \) Pa s is the reference viscosity at the bottom of the box, \( \rho_0 \approx 3300 \) kg m\(^{-3}\) and \( \theta_0 \approx 1300^\circ C \) are, respectively, the reference density and the reference potential temperature of the fluid, \( E \) is a variable activation energy, and \( V = 4.1 \times 10^{-6} \) m\(^3\) mole\(^{-1}\) is the activation volume (corresponding to a change in viscosity by a factor \( e \) over 100 km when \( \theta = \theta_0 \)). Other physical parameters in the model are the thermal diffusivity \( \kappa = 8 \times 10^{-7} \) m\(^2\) s\(^{-1}\), the thermal expansivity \( \alpha = 3.5 \times 10^{-5} \) K\(^{-1}\), and the heat capacity \( c_p = 1000 \) J kg\(^{-1}\) K\(^{-1}\).

Flow in the box is driven by a horizontal velocity \( U = 2.7 \times 10^{-9} \) m s\(^{-1}\) (\( \approx 8.6 \) cm yr\(^{-1}\)) at the upper surface of the box, and by a Gaussian potential temperature anomaly \( \theta_p(x, y) \) of radius \( a \) and amplitude \( \Delta \theta \) at the bottom. The bottom and the upstream and downstream ends of the box are open. The model requires a thermal boundary condition \( \theta = \theta_b(z) \) where the plate-driven flow enters the upstream end of the box. A natural choice is an error function.
the plume radius, using boxes of different sizes and varying where the precise form of the geotherm for this case is model-dependent, we assume for simplicity an error function geotherm whose scale depth $\delta$ can be varied to change the effective lithospheric thickness.

The equations to be solved are those governing conservation of mass, momentum and energy in a variable-viscosity fluid without inertia [5]. For simplicity, we use a Boussinesq formulation in which adiabatic temperature changes and the heat of fusion are neglected. The resulting temperatures must therefore be considered as potential temperatures. The numerical method used is that of Ribe and Christensen [5], with the addition of algorithms for calculating the melting rate and the advection of the depletion field. We use the melting parameterization of Watson and McKenzie [11], which specifies the mass fraction $X(p, T)$ of melt present under equilibrium (batch melting) conditions at pressure $p$ and temperature $T$. We calculate $T$ by correcting the model potential temperature $\theta$ for the effects of adiabatic compression and latent heat [11]. The local melting rate $\Gamma$ is:

$$\Gamma = \rho_0 \max \left( 0, \frac{DX}{Dt} \right)$$

where $D/Dr$ is the time rate of change following the motion (convective derivative). $\Gamma$ is non-negative because we assume that all melt is extracted instantaneously from the unmelted residuum, thereby preventing in situ refreezing [17]. Depletion of the solid is thus permanent, and can never decrease along a pathline. The depletion $F$ satisfies [17]:

$$\frac{DF}{Dt} = \frac{\Gamma}{\rho_0}, \quad 0 \leq F \leq 1$$

The effect of depletion on buoyancy is included in the equation of state:

$$\rho = \rho_0 \left[ 1 - \alpha (T - T_0) - \beta F \right]$$

where $\beta = 0.07$ [18].

We performed a total of 45 numerical experiments, using boxes of different sizes and varying the plume radius $a$, excess potential temperature $\Delta \theta$, thermal scale length $\delta$, plate speed $U$, and depletion buoyancy parameter $\beta$. Experiments were done both with a low activation energy $E = 2.0 \times 10^5$ J mole$^{-1}$ and a higher value $E = 3.2 \times 10^5$ J mole$^{-1}$. The latter is appropriate for olivine undergoing Newtonian diffusion creep [22], and corresponds to a decrease in viscosity by a factor $e$ over a temperature interval of 64°C when $\theta = 1300$°C and $z = 100$ km. Most experiments were done in a box of horizontal dimensions 1152 km $\times$ 2304 km with a uniform horizontal grid spacing $\Delta x = \Delta y = 9$ km and a variable vertical grid spacing $2 \text{ km} \leq \Delta z \leq 10$ km.

To achieve stationary solutions rapidly, we employed a damped iteration with a time step much larger than the Courant time step. For each experiment, we calculated the total melt production rate:

$$M = \int \Gamma dV$$

where the volume integral is over the whole model box. We calculated the surface topography anomaly by equating the vertical normal stress to the sum of the isostatic weight of the topography and the bending stresses in a thin elastic plate with flexural rigidity $D = 1.7 \times 10^{23}$ N m [23]. We then subtracted the topography predicted by the same model with the plume ‘turned off’ to obtain the uplift due to the plume alone. Topography due to volcanic loading and the lithospheric flexure it produces was neglected.

3. Inferences from scaling laws

In our judgement, there are three robust observations which constrain dynamical models of the Hawaiian plume. The first is the maximum uplift $H$ of the seafloor. Fig. 2a shows the uplift from southeast to northwest along the axis of the Hawaiian swell (squares), estimated by removing the effects of thermal cooling and volcanic loading [24]. The topography increases rapidly from southeast to northwest to a maximum value $H \approx 1350 \pm 100$ m [23,24], and subsequently decays at a slower rate.

The second constraint is the lateral extent of the Hawaiian swell. Fig. 2b shows contours of the swell topography after removal of spherical harmonic degrees $l \leq 12$. Let $W$ be the half-width of the topog-
Fig. 2. (a) Squares: estimates of uplift along the axis of the Hawaiian swell [4,24]. Solid and dashed lines: uplift predicted by the reference model with and without depletion buoyancy, respectively. (b) Light lines: contours of Hawaiian swell topography (spherical harmonic degrees $l \geq 13$). Solid and dashed lines: maximum lateral extent of plume thermal anomaly predicted by the reference model with and without depletion buoyancy, respectively. Dot is directly above the point where the local melting rate is maximum.

The third constraint is the total rate of melt production $M$. The rate required to produce both the Hawaiian islands and the thickened crust beneath them is $M \approx (1.4 \pm 0.16) \times 10^4$ kg s$^{-1}$ [25].

A fourth observable often used to constrain models of hotspot swells is the geoid to topography ratio (GTR), which for long wavelengths is proportional to the average depth of the low-density material compensating the swell [24]. However, Cserepes et al. [2] show that estimates of the GTR are strongly contaminated by volcanic loading and lithospheric flexure. We shall therefore not use the GTR as a constraint on our model.

In this section, we determine the set of all possible models that predict values of $M$, $H$ and $W$ within the ranges given above. We do this by determining complete scaling laws for the observables $M$, $H$ and $W$ as functions of the independent variables on which they depend. In our model there are four such variables: the thickness $z_l$ of the lithosphere above the plume stem, and the plume’s maximum potential temperature $\theta_i$, thermal buoyancy flux $B$, and minimum viscosity $\eta_p$. We defined $z_l$ as the depth (measured over the point where the local melting rate $I^*$ is maximum) to the streamline separating plume material from material that entered the box from upstream. The value of $z_l$ so defined includes any effect of plume-induced lithospheric thinning. We define $\theta_i$ to be the potential temperature at which the hottest upwelling material reaches the solidus of McKenzie and Bickle [7]. The thermal buoyancy flux $B$ is [26]:

$$B = -\rho_0 \alpha \int w(\theta - \theta_0) \, dS$$

where $w$ is the vertical velocity (positive downward) and the surface integral is over the bottom of the model box.

The scaling laws are determined in Appendix A, and have the forms:

$$M = B \, fct(\theta_i, z_l, \eta_p, \beta),$$
$$H = fct(\theta_i, z_l, B, \eta_p, \beta, U),$$
$$W = fct(\theta_i, B, \eta_p, \beta, U).$$

For generality we treated the depletion buoyancy parameter $\beta$ and the plate speed $U$ as additional independent variables even though we fix their values when applying the model to Hawaii. The RMS errors of the three scaling laws are 6.0%, 1.8%, and 2.5%, respectively. Their application saves a factor of $10^9$ in computer time relative to a full 3D computation.

We now use the scaling laws to determine the values of the unknown quantities $\theta_i$, $z_l$, $B$, and $\eta_p$ that are consistent with the acceptable ranges of
M, H, and W. The three observable constraints allow three of the unknowns to be expressed in terms of the fourth, which we choose to be $\theta_i$. We assume that $\theta_i > 1500^\circ$C, because a colder plume can only produce sufficient melting if the lithosphere is thinner than the depth of the deepest earthquakes below Hawaii (60 km [27]). An upper bound on $\theta_i$ is harder to establish; here we assume $\theta_i < 1600^\circ$C. The solid lines in Fig. 3 show the values of $B(\theta_i)$, $z_i(\theta_i)$, and $\eta_p(\theta_i)$ required to predict simultaneously the best estimates of $M$ ($1.4 \times 10^4$ kg s$^{-1}$), $H$ (1350 m), and $W$ (600 km). The dashed lines in each panel show the maximum allowable range of the variable in question ($z_i$, $B$, or $\eta_p$) consistent with the ranges of $M$, $H$, and $W$ given above.

Fig. 3 yields useful bounds on $z_i$, $B$, and $\eta_p$ (still assuming that 1500$^\circ$C $\leq \theta_i \leq 1600^\circ$C). First, the lithospheric thickness $z_i$ cannot exceed 89 km without reducing the melting rate $M$ too much. Second, the thermal buoyancy flux must satisfy 2200 kg s$^{-1} \leq B \leq 3500$ kg s$^{-1}$ to fit the cross-sectional area ($\sim HW$) of the swell. Finally, the minimum viscosity must satisfy $5 \times 10^{17}$ Pa s $\leq \eta_p \leq 1.3 \times 10^{19}$ Pa s. Because the spreading of the plume head is controlled by its own viscosity rather than that of the ambient mantle, lower or higher values of $\eta_p$ produce swells with $W/H$ too small and too large, respectively.

The scaling law for the melting rate also determines how much of the Hawaiian swell is compensated by depletion buoyancy. The importance of depletion buoyancy is measured by the ratio of the depletion buoyancy flux $\propto \beta M$ to the total (thermal plus depletion) buoyancy flux, or:

$$f = \frac{\beta M}{B + \beta M} \tag{8}$$

Because $M$ is proportional to $B$ by Eq. 7, $f$ is independent of $B$ and depends only on $\theta_i$, $z_i$, $\eta_p$ (assuming $\beta = 0.07$). For all values of these variables within the allowable ranges (Fig. 3), $0.20 < f < 0.33$, the most probable value being $f \approx 0.27$. Depletion buoyancy therefore plays a secondary role in the compensation of the Hawaiian swell, which is supported primarily by thermal buoyancy.

4. Reference Hawaiian plume model

Fig. 3 allows us to choose a reference model which predicts correctly the total melting rate $M$ and the amplitude $H$ and width $W$ of the Hawaiian swell. Because higher values of $z_i$ are more realistic for the old ($\sim 100$ Ma) lithosphere beneath Hawaii, we choose a model at the hotter end of the range in Fig. 3, with $\theta_i = 1593^\circ$C. However, models with other values of $\theta_i$ exhibit similar behavior. For the reference model, $z_i = 86$ km, $B = 3060$ kg s$^{-1}$, and $\eta_p = 8.0 \times 10^{17}$ Pa s (the activation energy is $E = 3.3 \times 10^5$ J mole$^{-1}$). These values are shown as dots in Fig. 3. This model predicts $M = 1.35 \times 10^4$ kg s$^{-1}$, $H = 1360$ m, and $W = 590$ km, respectively, 4% below, 1% above, and 1.5% below the best estimates of these quantities. The depletion buoyancy flux is $\beta M = 945$ kg s$^{-1}$, or 24% of the total (thermal plus depletion) buoyancy flux $B_{\text{tot}} = 4000$ kg s$^{-1}$. The
predicted axial uplift and outer limit of the thermal anomaly are shown by the solid lines in Fig. 2a and b, respectively. The reference model predicts well the rapid increase in uplift and its subsequent slow decrease from southwest to northwest along the swell axis.

Fig. 4 shows images in horizontal planes of the potential temperature $\theta$ (110 km depth), the depletion $F$ (100 km depth), and the melting rate $\Gamma$ (110 km depth). The point $(x, y) = (0, 0)$ is the center of the plume at 400 km depth. The plate motion is to the right. The thermal anomaly (Fig. 4a) widens downstream due to the combined effects of lateral gravitational spreading and advection by the moving plate [5]. The depletion anomaly (Fig. 4b) widens similarly, but is narrower than the thermal anomaly because of its lower buoyancy flux (see below). The general downstream decrease of $F$ near $y = 0$ at the fixed depth shown is due to progressive shallowing of the buoyant depleted material. The horizontal streaks in the potential temperature and depletion fields at $y = 240$ km reflect a small-scale convective instability of the lowermost lithosphere [6,28], which advects downward undepleted material (white) from the lowermost lithosphere. More than 99% of the depleted material originates in a primary melting

![Fig. 4. Images in horizontal planes of (a) potential temperature (110 km depth), (b) depletion (100 km depth) and (c) melting rate (110 km depth) for the reference model. Horizontal streaks at the right of images (a) and (b) are due to small-scale convective instability (see text).](image-url)
zone about 250 km long and 200 km wide above the plume stem (Fig. 4c). Minor additional melting occurs in a secondary melting zone located 300–550 km downstream from the center of the primary melting zone.

The structure of the double melting region is more clearly seen in Fig. 5, which shows images of $\theta$, $F$ and $\Gamma$ in the symmetry plane $y = 0$. The heavy lines are selected particle trajectories for the flow of the solid (melt trajectories are not calculated in this paper). The origin of the double melting zone can be understood by following a material particle along the lowermost/rightmost streamline in Fig. 5c. This material first undergoes pressure-release melting in the distance range $100 \text{ km} < x < 190 \text{ km}$. From 190 km to 320 km, the material moves deeper, shutting off melting. Finally, between 320 and 520 km the material reascends and melts again. No melting occurs beyond $x = 520 \text{ km}$ because the material has cooled below its solidus.

The physical mechanism responsible for the double melting region is a transition between two limiting flows: an axisymmetric flow near a rising cylindrical plume stagnating against a rigid lithosphere, and a ‘thin layer’ flow further downstream in which buoyant plume material spreads laterally. The flow.
near the plume stem can be modelled as that due to a density anomaly $-\delta \rho$ confined within a vertical cylinder $r \leq a$, $0 < z \leq b$ in a half-space $z > 0$ of fluid with constant viscosity beneath a rigid surface at $z = 0$. The flow is determined by representing the buoyant cylinder as a distribution of point forces [29] and integrating numerically. The resulting pattern of streamlines is shown in Fig. 6 for $b = 6a$. Material particles first ascend towards the surface $z = 0$, and then move down again. This behavior is due to the rigidity of the lithosphere, and vanishes if a free surface is assumed.

Further downstream, however, the buoyant plume material forms a thin layer that spreads laterally along the base of the lithosphere due to its intrinsic buoyancy. To conserve mass, the spreading layer must become thinner. Lubrication theory [4,5] shows that the (upward) vertical velocity of this thinning is:

$$w \approx -\left(\frac{Q^2 U^4 \eta_0}{g \delta \rho}\right)^{1/5} x^{-6/5}$$

where $Q$ is the volume flux of the plume. This vertical velocity is responsible for the melting in the secondary melting zone.

5. Topography and geoid anomalies

Depletion buoyancy affects significantly both the topography anomaly and the geoid-to-topography ratio (GTR). The solid lines in Fig. 2 show the axial uplift (Fig. 2a) and lateral extent of the plume material (Fig. 2b) for the reference model. The dashed lines show the same quantities for the reference model with depletion buoyancy "turned off" (i.e. with the same thermal buoyancy flux and excess potential temperature but with $\beta = 0$). Depletion buoyancy contributes little uplift upstream of the plume stem ($x < 0$), but about 350 m downstream ($x > 200$ km). But because the depletion anomaly is narrower than the thermal anomaly (Fig. 4), it has a negligible effect on the width of the swell (Fig. 2b).

The effect of depletion buoyancy on the GTR is best measured by comparing the reference model with another having no depletion buoyancy, but a larger thermal buoyancy flux ($B = 7450$ kg s$^{-1}$) so that it predicts the same maximum uplift ($H = 1360$ m) as the reference model. The GTR for both models is calculated by least-squares regression over the surface of the model box. The GTR for the model without depletion buoyancy is 0.0096, whereas that for the reference model was 0.0080, or 16% lower. This can be understood in terms of lubrication theory, which predicts that the thickness $S$ of the plume head scales as $B^{1/4} \delta \rho^{-1/2}$, where $B_{tot}$ is the total (thermal plus depletion) buoyancy flux [5]. Now depletion buoyancy both increases the density anomaly $\delta \rho$ and decreases the buoyancy flux $B_{tot}$ required to produce a given uplift (from 7450 kg s$^{-1}$ to 4000 kg s$^{-1}$ for the models referred to above). Both of these effects decrease $S$, thereby reducing the mean depth of the density anomalies and the GTR.

In summary, depletion buoyancy reduces the GTR significantly, but not enough to explain the low values (0.004–0.006) estimated from observations over the Hawaiian swell [30,31]. However, Cserepes et al. [2] show that the earlier estimates for the Hawaiian swell are probably biased too low by incomplete removal of the effects of the volcanic islands and lithospheric flexure, and that the GTR of our reference model ($= 0.0080$) is compatible with that of the Hawaiian swell proper.
6. Temporal evolution of Hawaiian volcanism

Individual Hawaiian volcanoes evolve through four petrologically defined stages [32–34]: (1) an alkalic ‘preshield’ stage, represented today by Loihi seamount; (2) a tholeiitic ‘shield’ stage, during which 99% of the volcano’s volume is erupted; (3) a ‘postshield’ alkalic stage representing about 1% of the total volume; and (4) after a hiatus in volcanic activity of variable length (0.25–2.5 Ma), a ‘rejuvenated’ stage that produces small volumes of silica-poor lavas. Fig. 7 shows the durations of the last 3 stages for 12 Hawaiian volcanoes. The preshield stage is not shown because it is (presumably) buried beneath the shield volcanics and therefore not observable.

The melt extraction rates predicted by our model can be compared with the temporal evolution of Hawaiian volcanism shown in Fig. 7. We assume that all melt formed at depth is extracted vertically. The rate of melt supply to the crust is then:

$$q(x, y) = \int \Gamma \, dz$$

(10)

Fig. 8a shows $q(x, y)$ for the reference model. At any time, melt is being supplied to the crust over two roughly elliptical regions corresponding to the primary and secondary melting zones (Figs. 4 and 5). The width of the primary melt extraction region is $\sim 180$ km, somewhat less than the width ($\sim 250–300$ km) of thickened crust beneath the Hawaiian islands inferred from seismic refraction data [35].

To transform the supply rate $q(x, y)$ into a (volumetric) volcanic eruption rate $Q_e(t)$, we follow a material point on the lithosphere (i.e. a volcano) as it moves at a speed $U = 8.6$ cm yr$^{-1}$ from left to right. We assume that the instantaneous eruption rate at this volcano is the integral of the supply rate over a circle $C(r)$ of radius $r$ centered on the volcano’s current position:

$$Q_e(t) = \rho_c^{-1} \int_{C(r)} q(x + Ut, y) \, dx \, dy$$

(11)

where $\rho_c = 2700$ kg m$^{-3}$ is the crustal density. The radius $r$ ($= 33.6$ km) is determined by requiring the volume of lava erupted while the volcano is over the primary melting zone to be $92,000$ km$^3$, the volume of the first three stages of Hawaiian volcanism [36]. The resulting curve of $Q_e(t)$ is shown as a dashed line in Fig. 8b. Values of $Q_e$ above the secondary melting zone have been multiplied by 100 for visibility. The three rectangles at lower left show estimates of the durations and volumes of (from left to right) the preshield, shield, and postshield stages of Hawaiian volcanism [36]. The horizontal lines indicate the duration of rejuvenated stage volcanism on Niihau and Kauai, the only two Hawaiian islands that exhibit the entire 5–6 Ma eruption history [34,37]. The duration shown for Kauai includes all K–Ar ages represented by more than one sample [37]. The arrow at right shows an independent estimate of the eruption rate (also multiplied by 100) for Hawaiian rejuvenated-stage volcanism [38].

Fig. 8 suggests a new dynamical explanation for the four stages of Hawaiian volcanism. We propose that the first three stages (preshield, shield, and postshield) occur as the volcano passes over the primary melting zone, and the final rejuvenated stage as it passes over the secondary melting zone. The major features of the predicted eruption rate curve $Q_e(t)$ are consistent with this interpretation. Above the primary melting zone, the curve is roughly symmetric.
about its maximum, in qualitative agreement with the timing and relative volumes of the first three stages. Eruption ceases between the postshield and rejuvenated stages, as observed, and the predicted timing of the rejuvenated stage agrees well with that observed on both Kauai and Niihau. Finally, the maximum eruption rate predicted for the rejuvenated stage is within 7% of Walker’s estimate [38].

We close this section by determining the source of the material which undergoes melting. Material enters the model box both through the upstream end (with the shear flow) and through the bottom (with the plume). The source of the material currently at a point where melting is occurring (i.e. \( \Gamma > 0 \)) can be determined by tracing backward the streamline passing through that point until a boundary of the model box is reached. We thereby find that all the material in both melting zones originally entered the box from below, through a circle of radius 30 km in the hot central core of the plume stem. No direct melting of either the ambient upper mantle or of the lithosphere occurs. However, subsequent ‘indirect’ melting of the lithosphere may occur as melt migrates through it.

7. Discussion

A principal result of this study has been to quantify the importance of melting-induced ‘depletion buoyancy’ in plume dynamics. Phipps Morgan et al. [15] proposed that the Hawaiian swell is compen-
sated primarily by depletion buoyancy rather than thermal buoyancy. Our scaling laws, however, show that the depletion buoyancy flux cannot exceed 33% of the total (thermal plus depletion) buoyancy flux, and in our reference model the fraction is only 24%. Depletion buoyancy therefore plays a secondary (albeit significant) role in the compensation of the Hawaiian swell, which is supported primarily by thermal buoyancy. Note that our depletion buoyancy parameter $\beta = 0.07$ is even larger than that ($\beta = 0.06$) assumed by Phipps Morgan et al. [15].

Our scaling laws imply that the thermal buoyancy flux $B$ of the Hawaiian plume lies in the range $2200\text{--}3500\text{ kg s}^{-1}$ (Fig. 3b), much less than values estimated previously from the cross-sectional area of the Hawaiian swell (6300 kg s$^{-1}$ [26]; 8700 kg s$^{-1}$ [39]). One reason the earlier estimates are too high is that they include the depletion buoyancy flux. However, this alone cannot explain the discrepancy, because the latter flux is only $\beta M \approx 870 - 1100$ kg s$^{-1}$. The more important reason is that both [26] and [39] assumed that the average downstream velocity $U_{sw}$ of the low-density plume material is equal to the plate speed $U$. In reality, however, $U_{sw}/U \approx 0.6 - 0.7$, due to the effect of asthenospheric shear [5]. Because buoyancy flux is proportional to the cross-sectional area of the topography times $U_{sw}$, assuming $U_{sw} = U$ leads to an overestimate (by a factor $U/U_{sw}$) of the buoyancy flux required to produce a given topography.

The lithospheric thickness required by our scaling laws ($z_l < 89$ km for $\theta_i < 1600\degree$C) is lower than that predicted by a half-space cooling model, but in good agreement with models that parameterize convective heat transport to the bottom of the oceanic lithosphere [20,21]. However, because our estimate of $z_l(\theta_i)$ is constrained primarily by the observed melting rate $M$, it will be affected by any uncertainties in the melting parameterization used (see below).

Our scaling laws also demonstrate that the minimum viscosity in the plume head must exceed $5 \times 10^{17}$ Pa s (Fig. 3c). This result has important implications for the dynamical regime of the Hawaiian plume. The spreading head of a thermal plume undergoes two transitions as the viscosity contrast $\gamma$ between the ambient mantle and the plume increases: an initially smooth steady-state structure is replaced first by a more complex steady pattern with roll-like convective instabilities, and eventually becomes highly time-dependent [6]. All our numerical experiments that satisfy the observational constraints fall within the second of these regimes (steady with roll-like instabilities), and have $\gamma \leq 90$ (the reference model has $\gamma = 84$). The time-dependent regime can be attained by reducing the plume viscosity $\eta_p$ to increase $\gamma$, but the resulting models would not predict acceptable values of $H$ and $W$. This suggests that the Hawaiian plume is probably in the intermediate steady-state regime.

Our reference model yields surprisingly good predictions of the eruption rate and timing of rejuvenated-stage Hawaiian volcanism. Moreover, the double melting zone is a robust feature that occurs in all 45 of our numerical experiments. The origin of Hawaiian rejuvenated volcanism has long been puzzling. Two main hypotheses have been proposed to explain it: a rapid change from subsidence to uplift as the volcano overrides the flexural arch created by an adjacent younger shield [34,40], and melting of the lower lithosphere by conductive heating downstream from the hotspot [41]. However, the uplift model cannot explain why Hawaiian volcanoes remain active for 5$-$6 Ma, and the conductive heating model cannot explain why the activity first stops and then starts again. Our model shows that both of these features arise naturally from the fundamental fluid mechanics of plume/lithosphere interaction.

The most obvious way to improve our model is to incorporate a thermodynamically realistic fractional melting parameterization. Unfortunately, this is currently impossible due to the lack of experimental constraints at high pressures (M. Hirschmann, pers. commun., 1998). Recent work suggests that the principal effect of fractional melting is to decrease the 'productivity' of mantle upwelling, i.e. the amount of melt produced per unit pressure decrease [42]. A model including fractional melting would thus require a hotter plume (higher $\theta_i$) and/or a thinner lithosphere (smaller $z_l$) to yield the same total melting rate $M$. Fractional melting will also affect the secondary melting zone, where the material will already have lost some of its fusible components during previous melting near the plume stem. However, our model predicts that the degree ($X$) of this previous melting is less than that of the subsequent secondary melting. The secondary melting zone will...
therefore persist even in the presence of fractional melting, although it may be weakened somewhat.

There remain several features of Hawaiian volcanism that our model does not explain. First, the predicted duration (~2.2 Ma) of volcanism over the primary melting zone is longer than the estimated duration of 1.4 Ma [36]. A possible reason is that melt transport through the lithosphere is shut off after a certain time by the compressive flexural stresses generated by the weight of the volcanic edifice itself [43,44]. Second, our model does not account for the variable timing and duration of rejuvenated stage volcanism [34]. Predicting this variability probably requires a more realistic model with time-dependent lubrication theory models for plume–plate interaction [4,5].

Finally, we have made no attempt here to explain the characteristic petrological, geochemical, and isotopic evolution of Hawaiian lavas. This will be the subject of future work.

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Appendix A. Scaling laws for \( H, W \) and \( M \)

A.1. Maximum uplift \( H \)

The scaling law for \( H \) was determined from 21 numerical experiments in a box of lateral dimensions 1152 km \( \times \) 2304 km. Lubrication theory models for plume–plate interaction [4,5] show that the thickness \( S \) of the plume material depends only on the plate speed \( U \), the plume’s volume flux \( Q \), and \( \sigma = g \rho b / 48 \eta_p \), where \( \rho_b \) and \( \eta_p \) are the plume’s density anomaly and interior viscosity, respectively. Moreover, isostasy requires \( S \rho_b \approx H \Delta \rho \), where \( \Delta \rho = \rho_b - \rho_w \) and \( \rho_w \) is the density of seawater. Dimensional analysis then yields:

\[
H = \frac{\Delta \rho}{S \rho_b} \left( \frac{Q}{\sigma} \right)^{1/4} G(\Pi_b), \quad \Pi_b = \frac{Q \sigma}{U^2}
\]

where \( \Pi_b \) is the ‘buoyancy number’ and \( G \) is a function to be determined. The total effective density anomaly is:

\[
\Delta \rho = \rho_0 [a \Delta \theta_i + C_1 \beta X_{\text{max}}]
\]

where \( X_{\text{max}} \) is the maximum melt fraction above the plume stem and \( C_1 \) is a constant weighting factor. We calculate \( \sigma \) using the minimum viscosity in the plume head, and the total volume flux is:

\[
Q = (B + \beta M) / \Delta \rho
\]

Fig. 9 shows a plot of \( G \) vs. \( \Pi_b \) for the 21 numerical experiments, with \( C_1 = 0.52 \). The points are well represented by:

\[
G(\Pi_b) = 1.26 \tanh (1.01 \Pi_b + 0.687)
\]

shown by the solid line in Fig. 9. The last step is to determine a scaling law for \( X_{\text{max}} \). Let \( z_i, p_i = \rho_0 g z_i, \theta_i \) and \( T_i \) be, respectively, the depth, pressure, potential temperature, and real temperature at the point where \( X = X_{\text{max}} = X(p_i, T_i) \). The numerical experiments show that \( z_i - z_i = 10.4 \pm 1.3 \) km. Further, \( \theta_i \) is found to depend only on \( \theta_i \) and \( \Pi_b \) as:

\[
\frac{\theta_i - \theta_i}{\theta_i - \theta_i} = \frac{2.0}{\Pi_b + 2.0}
\]

The temperature \( T_i \) is found by correcting \( \theta_i \) for the effects of adiabatic decompression and latent heat according to:

\[
T_i - T_i(p_i) = K(\theta_i - \theta_i(p_i), p_i)
\]
where $T_s(p)$ is the solidus temperature as a function of pressure [7], $\theta_s(p) = T_s(p) \exp(-gazT/c_p)$ and $K(\theta, p)$ is given by eq. 15 of [11] with the coefficients $\alpha_{\text{iso}}$ calculated for an entropy of melting $\Delta S = 400 \text{ J kg}^{-1} \text{ K}^{-1}$ [48]. The RMS error of the above scaling law is 1.8%.

A.2. Swell width $W$  

The scaling law for $W$ was determined from 13 numerical experiments in a box of lateral dimensions $1600 \text{ km} \times 3200 \text{ km}$. We define the width of the swell as the outermost limit of the plume’s thermal anomaly, i.e. the zero contour of the ‘isostatic topography’:

$$h_{\text{iso}}(x, y) = \frac{\rho_0 a}{\Delta \rho} \int_0^h \delta \theta \, dz$$  \hspace{1cm} (18)

where $\delta \theta(x, y, z)$ is the plume’s potential temperature less that of the same model with the plume ‘turned off’. Because the depletion anomaly is always narrower than the thermal anomaly, the location of the zero contour of $h_{\text{iso}}$ is nearly independent of $\beta$. We therefore need only consider the case $\beta = 0$.

Let $W$ be the width of the zero contour of $h_{\text{iso}}$ at the hotspot location $x = x_0$. The thermal buoyancy flux of the plume must equal the flux of topographic buoyancy advected downstream, or:

$$Q \delta \rho \approx UWH \Delta \rho$$  \hspace{1cm} (19)

Using Eq. 12 and rearranging, we find:

$$W = C_2 \frac{Q^{3/4} a^{1/4}}{UG \Delta \rho}$$  \hspace{1cm} (20)

where $C_2$ is a constant and $\delta \rho$ and $Q$ are calculated from Eqs. 13 and 14 with $\beta = 0$. Least-squares fitting of the results of the 13 numerical experiments yields $C_2 = 1.29 \pm 0.032$. The zero contours of $h_{\text{iso}}$ (Fig. 10, top), when scaled by $W$, cluster closely around a single universal curve (bottom). The curve closest to the middle of the range is fit with an RMS error of 0.006 for $x/W \leq 2.5$ by:

$$\frac{y}{W} = 1.37 \cos^{-1} \left( \frac{1}{0.474/x + 1.342} \right)$$  \hspace{1cm} (21)

The accuracy of our ‘isothermal’ scaling law for $W$ suggests that the spreading of the plume material is little affected by its cooling. This is demonstrated by a simple scaling argument, suggested by N. Sleep (pers. commun., 1999). Because the plume head has characteristic thickness $(Q/\sigma)^{1/4}$ [5], its cooling time is $t_{\text{cool}} \approx (Q/\sigma)^{1/2} \cdot \text{K}^{-1}$. The time required for a layer of viscous fluid emitted from a point source to reach a characteristic lateral dimension $W \approx Q^{3/4} \sigma^{1/4} / U$ is $t_{\text{spread}} \approx Q^{3/4} \sigma^{1/4} / U^2$. The ratio of these times is an effective Peclet number:

$$\frac{t_{\text{cool}}}{t_{\text{spread}}} = \text{Pe} = \frac{U^2}{Q^{1/4} \sigma^{3/4} \Delta \rho K}$$  \hspace{1cm} (22)

Spreading of the plume head is unaffected by cooling if $\text{Pe} \gg 1$, which is the case for most of our numerical experiments ($\text{Pe} = 33$ for the reference model).

Fig. 10. (Top) Zero contours of isostatic topography $h_{\text{iso}}$ (Eq. 18) for 13 numerical experiments with activation energy $E = 2.0 \times 10^5 \text{ J mole}^{-1}$ (solid lines) or $3.2 \times 10^5 \text{ J mole}^{-1}$ (dashed lines), various buoyancy fluxes $B$ and plate speeds $U$, and no depletion buoyancy ($\beta = 0$). (Bottom) Same, but scaled with the length scale $W$ (Eq. 20).

A.3. Total melting rate $M$

The scaling law for $M$ was determined from 32 numerical experiments in a box of lateral dimensions $1152 \text{ km} \times 2304 \text{ km}$. The melting rate scales as:

$$M \approx \int \Gamma dV \approx (a^2 \Delta x) c_p u \frac{dX}{dz}$$  \hspace{1cm} (23)

where $a$ is the plume radius, $\Delta x$ is the vertical extent of the melting zone, and $u$ is the vertical velocity. However, the buoyancy flux scales as $B \approx wa^2 c_p a(\theta_i - \theta_b)$, whence:

$$M \approx \frac{B}{a(\theta_i - \theta_b)} \frac{\Delta x}{\Delta z} \frac{dX}{dz} \approx \frac{B}{a(\theta_i - \theta_b)} F(\theta_i, \eta, \eta', \beta)$$  \hspace{1cm} (24)

Now the total derivative $dX/dz$ is the difference of two much larger numbers representing the opposed effects on $X$ of cooling ($\partial X/\partial T > 0$) and decompression ($\partial X/\partial p < 0$). An analytical scaling law for $dX/dz$ is therefore bound to be inaccurate. Better results are obtained by direct least-squares fitting of the function $F$, yielding:

$$F = a_0 + \sum_{n=1}^{4} a_n (x_{n0} + b_n x_0)$$  \hspace{1cm} (25)

where $x_0 = x_0$ (in Kelvin), $x_1 = z_0$ (in m), $x_2 = \log_{10} \eta_0$ (in Pa s), $x_3 = \beta$, $a_0 = -0.278$, $a_1 = 0.000525$, $a_2 = -7.44 \times 10^{-6}$.
\[ a_3 = -0.0144, a_4 = -0.0413, b_2 = 3.24 \times 10^{-11}, \text{ and } b_1 = b_3 = b_4 = 0. \] The RMS error of Eq. 25 is 6.0%. Warning: Eq. 25 is not valid beyond the ranges of the variables used in our numerical experiments: 1512°C \leq \theta \leq 1596°C, 68 km \leq z \leq 90 km, 7.2 \times 10^{12} \text{ Pa s} \leq \eta_p \leq 4.8 \times 10^{18} \text{ Pa s}, \text{ and } 0 \leq \beta \leq 0.07.

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